The Identity Matrix

1 $\mathbf{M} = \begin{pmatrix} \sqrt{12} & 4 \\ 0 & \sqrt{8} \end{pmatrix}$ $\mathbf{N} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\mathbf{MN} = \sqrt{6}\mathbf{I}$ where **I** is the identity matrix.

Work out the values of a, b, c, d. [6 marks]

Worked Solution

$$\sqrt{6}\mathbf{I} = \sqrt{6}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{6} & 0 \\ 0 & \sqrt{6} \end{pmatrix}$$

Multiply the Matrices together to get two pairs of simultaneous equations.

$$\begin{pmatrix} \sqrt{12} & 4 \\ 0 & \sqrt{8} \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \sqrt{12}a + 4c & \sqrt{12}b + 4d \\ \sqrt{8}c & \sqrt{8}d \end{pmatrix} = \begin{pmatrix} \sqrt{6} & 0 \\ 0 & \sqrt{6} \end{pmatrix}$$

Solve the equations for a, b, c, d

$$a\sqrt{12} + 4c = \sqrt{6} \qquad b\sqrt{12} + 4d = 0$$

$$c\sqrt{8} = 0 \qquad d\sqrt{8} = \sqrt{6}$$

Start by looking at the value of c

$$c\sqrt{8} = 0$$

$$\therefore c = 0$$

Now look at a

$$a\sqrt{12} + 4c = \sqrt{6}$$

$$a\sqrt{12} = \sqrt{6}$$

$$a = \frac{\sqrt{6}}{\sqrt{12}}$$

$$a = \sqrt{\frac{1}{2}}$$

$$a = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$a = \frac{\sqrt{2}}{2}$$

Now look at d

$$d\sqrt{8} = \sqrt{6}$$

$$d = \frac{\sqrt{6}}{\sqrt{8}}$$

$$d = \sqrt{\frac{3}{4}}$$

$$d = \frac{\sqrt{3}}{\sqrt{4}}$$

$$d = \frac{\sqrt{3}}{2}$$

Finally look at b

$$b\sqrt{12} + 4d = 0$$

$$b\sqrt{12} + \frac{4\sqrt{3}}{2} = 0$$

$$b\sqrt{12} + 2\sqrt{3} = 0$$

$$b\sqrt{4}\sqrt{3} + 2\sqrt{3} = 0$$

$$2\sqrt{3}b + 2\sqrt{3} = 0$$

$$2\sqrt{3}b = -2\sqrt{3}$$

$$b = -1$$

So $\mathbf{N} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -1 \\ 0 & \frac{\sqrt{3}}{2} \end{pmatrix}$